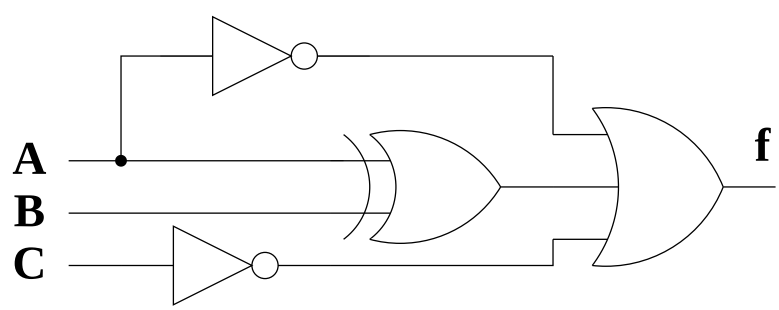
Embedded Real-Time Systems (AME 3623)
Homework 1 Solutions

February 8, 2008

Question 1

Consider the following circuit.



1. (10 pts) What is the corresponding truth table?

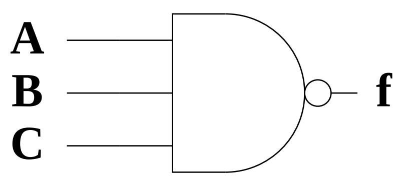
   Shortcut: f is true any time A or C are false.

A	B	C	f
0	0	0	1
0	0	1	1
0	1	0	1
0	1	1	1
1	0	0	1
1	0	1	1
1	1	0	1
1	1	1	0

2. (10 pts) Show the simplified circuit (this should require very little reduction).

   The short path: first design the circuit for $\bar{f}$ and then add a NOT to the end.

   This gives us:



The direct reduction approach:

$\bar{A}\bar{B}\bar{C} + \bar{A}\bar{B}C + \bar{A}B\bar{C} + \bar{A}BC + A\bar{B}\bar{C} + AB\bar{C} + A\bar{B}C$	start
$\bar{A}\bar{B}\bar{C} + \bar{A}\bar{B}\bar{C} + \bar{A}\bar{B}\bar{C} + \bar{A}\bar{B}C + \bar{A}\bar{B}C + \bar{A}B\bar{C} + \bar{A}B\bar{C} +$ $\bar{A}BC + A\bar{B}\bar{C} + A\bar{B}\bar{C} + AB\bar{C} + A\bar{B}C$	$X + X = X$
$\bar{A}\bar{B}\bar{C} + \bar{A}\bar{B}C + \bar{A}B\bar{C} + \bar{A}BC + \bar{A}\bar{B}\bar{C} + \bar{A}\bar{B}C + A\bar{B}\bar{C} +$ $A\bar{B}C + \bar{A}B\bar{C} + \bar{A}\bar{B}\bar{C} + A\bar{B}\bar{C} + AB\bar{C}$	Commutative Law
$\bar{A}\bar{B}(\bar{C} + C) + \bar{A}B(\bar{C} + C) + \bar{A}\bar{B}(\bar{C} + C) + A\bar{B}(\bar{C} + C) +$ $\bar{A}(B + \bar{B})\bar{C} + A(\bar{B} + B)\bar{C}$	Distributive Law
$\bar{A}\bar{B} + \bar{A}B + \bar{A}\bar{B} + A\bar{B} + \bar{A}\bar{C} + A\bar{C}$	$X + \bar{X} = 1$, and $X * 1 = X$
$\bar{A}(\bar{B} + B) + (\bar{A} + A)\bar{B} + (\bar{A} + A)\bar{C}$	Distributive Law
$\bar{A} + \bar{B} + \bar{C}$	$X + \bar{X} = 1$, and $X * 1 = X$
$\overline{ABC}$	DeMorgan's Law

Note: partial reduction (so that complexity is less than the original circuit) is worth partial credit.

Question 2

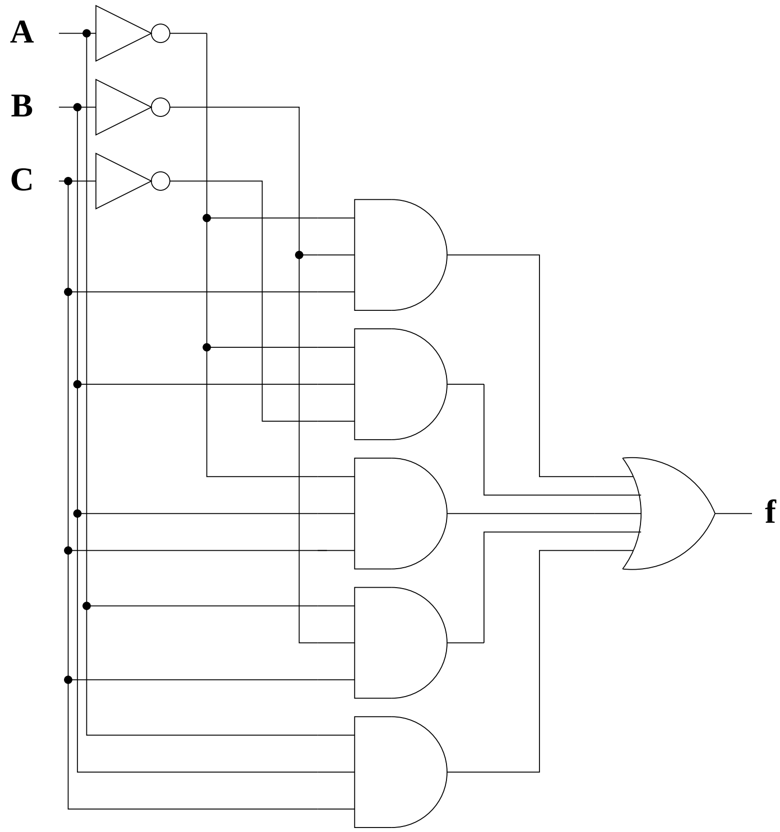
Consider the following function:

A	B	C	f
0	0	0	0
0	0	1	1
0	1	0	1
0	1	1	1
1	0	0	0
1	0	1	1
1	1	0	0
1	1	1	1

1. (10pts) Show the algebraic expression for the "minterm" form of the circuit (set of 3-term ANDs that are then ORed together).

$$f = \bar{A}\bar{B}C + \bar{A}B\bar{C} + \bar{A}BC + A\bar{B}C + ABC$$

2. (10pts) Show the corresponding circuit

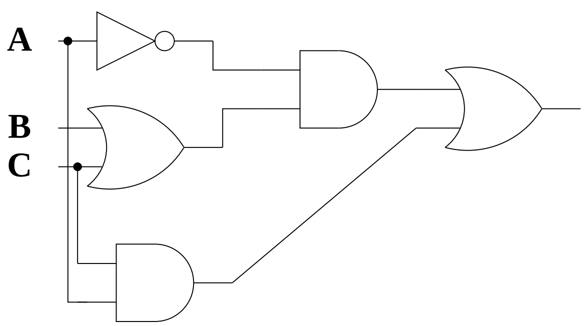


3. (10pts) Reduce this algebraic expression to a minimal form (note that there may be more than one correct answer). **Show each step, showing the name of the algebraic rule that you use.**

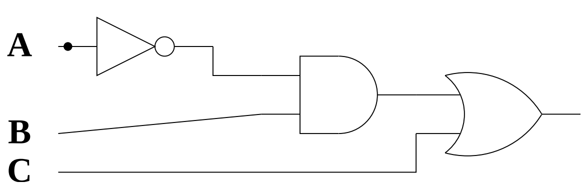
$$\begin{aligned}
\bar{A}\bar{B}C + \bar{A}B\bar{C} + \bar{A}BC + A\bar{B}C + ABC & \qquad \mathit{Start} \\
\bar{A}(\bar{B}C + B\bar{C} + BC) + A(\bar{B} + B)C & \qquad \mathit{AssociativeLaw} \\
\bar{A}(\bar{B}C + B\bar{C} + BC + BC) + A(\bar{B} + B)C & \qquad X + X = X \\
\bar{A}(\bar{B}C + BC + B\bar{C} + BC) + A(\bar{B} + B)C & \qquad \mathit{CommutativeLaw} \\
\bar{A}((\bar{B} + B)C + B(\bar{C} + C)) + A(\bar{B} + B)C & \qquad \mathit{AssociativeLaw} \\
\bar{A}(C + B) + AC & \qquad X + \bar{X} = 1; X * 1 = X \qquad (1) \\
\bar{A}C + \bar{A}B + AC & \qquad \mathit{DistributiveLaw} \\
\bar{A}B + \bar{A}C + AC & \qquad \mathit{CommutativeLaw} \\
\bar{A}B + (\bar{A} + A)C & \qquad \mathit{DistribuativeLaw} \\
\bar{A}B + C & \qquad X + \bar{X} = 1; X * 1 = X
\end{aligned}$$

Note: equation 1 is sufficient to receive full credit.

4. (10pts) Show the corresponding circuit



OR



Question 3

Consider the following function:

A	B	C	f
0	0	0	0
0	0	1	0
0	1	0	1
0	1	1	0
1	0	0	0
1	0	1	1
1	1	0	1
1	1	1	1

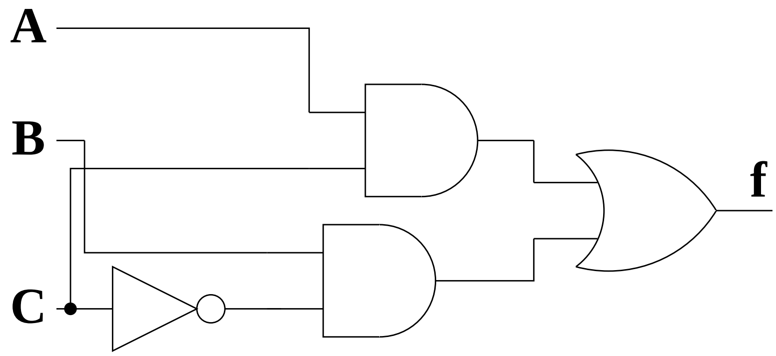
1. (10pts) Show the algebraic expression for the "minterm" form of the circuit.

   $\bar{A}B\bar{C} + A\bar{B}C + AB\bar{C} + ABC$

2. (10pts) Reduce this algebraic expression to a minimal form. Show each step, showing the name of the algebraic rule that you use.

$$\begin{aligned} &\bar{A}B\bar{C} + A\bar{B}C + AB\bar{C} + ABC && \\ &\bar{A}B\bar{C} + AB\bar{C} + A\bar{B}C + ABC && \mathit{Communative\ Law} \\ &(\bar{A} + A)B\bar{C} + A(\bar{B} + B)C && \mathit{Distributive\ Law} \\ &B\bar{C} + AC && X + \bar{X} = 1 \mathit{\ and\ } X * 1 = X \end{aligned} \tag{2}$$

3. (10pts) Show the reduced circuit.



Question 4

1. (10 pts) Suppose you need a circuit to perform an AND between two inputs, but that you only have 2-input NAND gates. What would the circuit look like? Show the algebraic rules that you use.

   In order to create an AND gate from a NAND, we need to invert the output of the NAND. So - the question is how to create an inverter from a NAND gate. The algebraic expression for our NAND gate is:

   $C = \overline{A * B}$

   We can wire inputs A and B to anything we like. In particular, we can choose: $B = A$. This gives us:

   $C = \overline{A * A}$

   Which simplifies to (by $X = X * X$):

   $C = \overline{A}$

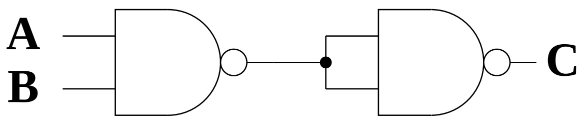
   Alternatively, we can choose $B = 1$ (i.e. a constant):

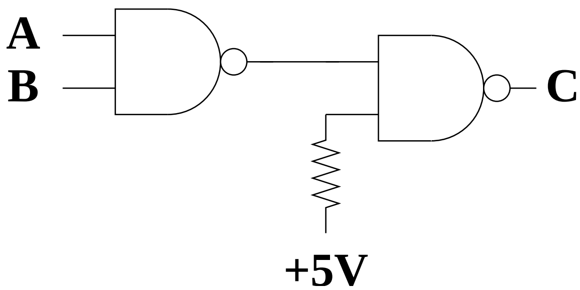
   $C = \overline{A * 1}$

   Which simplifies to (by $X = X * 1$):

   $C = \overline{A}$

The corresponding circuits are:





Note: it is acceptable for the latter answer to just have a logical '1' as the constant input into the 2nd NAND gate.

2. (10 pts) Suppose you need a circuit to perform an OR between two inputs, but that you only have 2-input NAND gates. What would the circuit look like? Show the algebraic rules that you use.

   DeMorgan's Law gives us the relationship between AND and OR:

   $A + B = \overline{\overline{A} + \overline{B}}$

In the previous question, we show how to use a NAND gate to implement an inverter. So - the resulting circuit is: