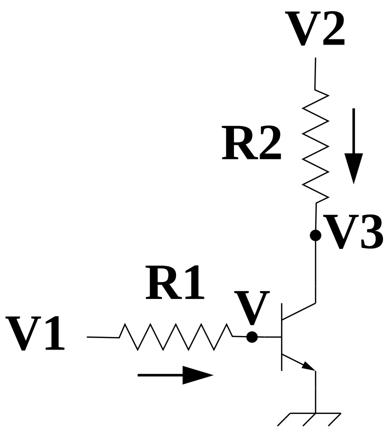
# Embedded Systems (CS [45]163)
# Homework 4 Solutions

May 5, 2010

## Question 1

Consider the following circuit:

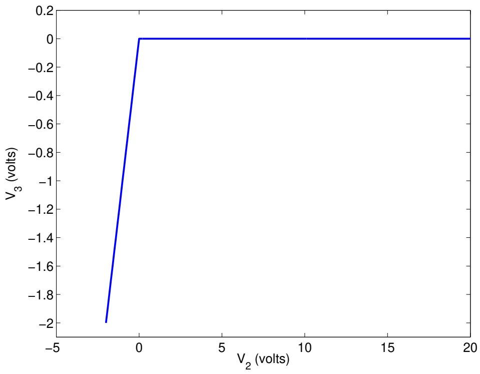


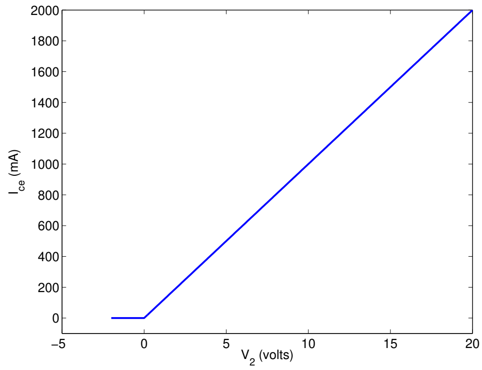
1. (5pts) What is always true no matter the state of the transistor? (i.e., what are the fundamental equations?)

$$\begin{aligned} I_2 &= I_{ce} \\ V_2 - V_3 &= R_2 I_2 \\ V_1 - V &= R_1 I_1 \\ I_1 &= I_{be} \\ 0 \le I_{ce} &\le g I_{be} \\ 0 &\le V_3 \end{aligned}$$

2. (15pts) Assume that $V1 = 5V$, $V_f = 0.7V$, $R1 = 2K\Omega$, $g = 10000$ and $R2 = 10\Omega$. Show $I_{CE}$, $V$ and $V3$ as a function of $-2 \le V_2 \le 20$.

   $V$ is always $V_f$.



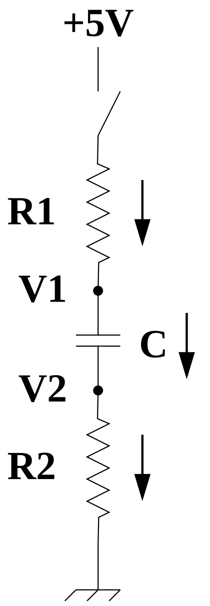
3. (10pts) Assume the same resistances as above and that $V2 = 20V$. What would $V1$ have to be such that $V3 = 15V$?

   In this case:

$$
\begin{aligned}
I_{ce} &= \frac{V2 - V3}{R2} \\
&= 0.5A \\
I_{be} &= I_{ce}/g \\
&= \frac{V_1 - V_f}{R1} \\
V_1 &= R_1 I_{ce}/g + V_f \\
&= 0.8V
\end{aligned}
$$

# Question 2

Consider the following circuit:




Assume that the switch closes at $t = 0$ and that $V1(0) = V2(0) = 0V$. Define $V = V_1 - V_2$

1. (10pts) What are the fundamental equations that determine the behavior of this circuit?

$$
\begin{aligned}
I_{R1} &= I_C \\
I_{R2} &= I_C \\
C\frac{d(V_1 - V_2)}{dt} &= I_C \\
V_2 - 0 &= R_2 I_{R2} \\
5 - V_1 &= R_1 I_{R1}
\end{aligned}
$$

(this latter equation is only true when the switch is closed)

2. (10pts) Derive an equation for $I_C$ in terms of $V$, $R_1$, and $R_2$.

$$\begin{aligned}
V_1 - 5 - V_2 &= -R_1 I_{R1} - R_2 I_{R2} \\
V_1 - V_2 - 5 &= -I_C(R_1 + R_2) \\
I_C &= \frac{5 - V}{R_1 + R_2}
\end{aligned}$$

3. (20pts) Derive an equation for $V(t)$. Hint: $V_1$ and $V_2$ should drop out of the equations.

$$\begin{aligned}
C\frac{dV}{dt} &= \frac{5 - V}{R_1 + R_2} \\
C(R_1 + R_2)\frac{dV}{5 - V} &= dt
\end{aligned}$$

Take $u = 5 - V$. Therefore: $du = -dV$.

$$C(R_1 + R_2)\frac{du}{u} = -dt$$

Integrate both sides:

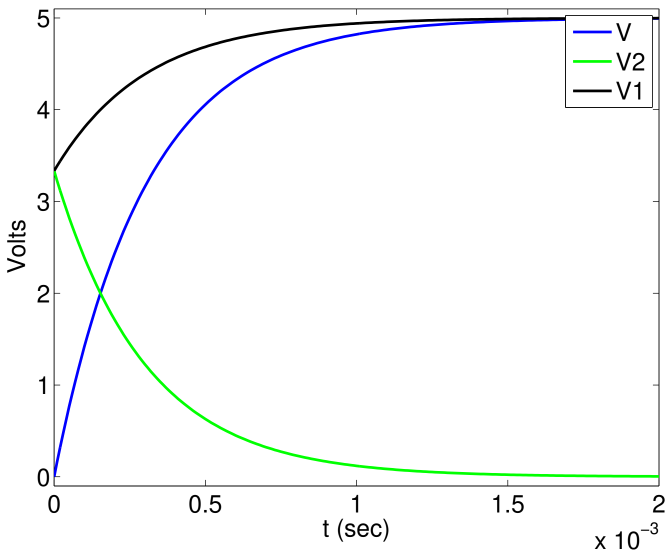
$$\begin{aligned}
\int_{u(0)}^{u(T)} C(R_1 + R_2)\frac{du}{u} &= \int_0^T -dt \\
C(R_1 + R_2)\int_{u(0)}^{u(T)} \frac{du}{u} &= -\int_0^T dt \\
C(R_1 + R_2) ln(u) \mid_{u(0)}^{u(T)} &= -t \mid_0^T \\
C(R_1 + R_2) ln\left(\frac{u(T)}{u(0)}\right) &= -T \\
ln\left(\frac{5 - V(T)}{5 - V(0)}\right) &= \frac{-T}{C(R_1 + R_2)} \\
\frac{5 - V(T)}{5 - V(0)} &= e^{-\frac{T}{C(R_1+R_2)}} \\
V(T) &= 5 - (5 - V(0))e^{-\frac{T}{C(R_1+R_2)}} \\
V(T) &= 5 - 5e^{-\frac{T}{C(R_1+R_2)}}
\end{aligned}$$

4. (10pts) Derive equations for $V_1(t)$ and $V_2(t)$.

$$
\begin{aligned}
V_2 &= R_2 I_{R2} \\
&= R_2 I_C \\
&= (5 - V)\frac{R_2}{R_1 + R_2} \\
V_1 &= 5 - R_1 I_{R1} \\
&= 5 - R_1 I_C \\
&= 5 - (5 - V)\frac{R_1}{R_1 + R_2}
\end{aligned}
$$

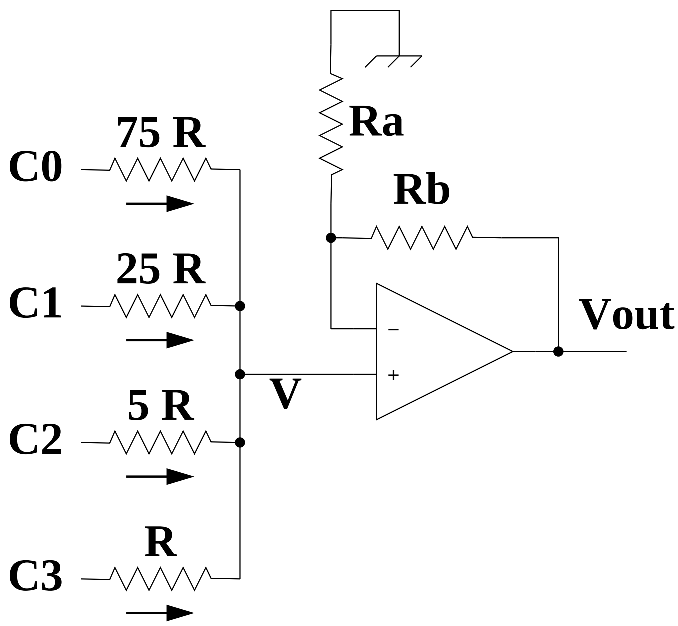
(and we already know $V(t)$)

For $C = 1\mu F$, $R1 = 100\Omega$, and $R2 = 200\Omega$:



## Question 3

Consider the following circuit:




1. (10 pts) What are the fundamental equations that determine $V$ and other associated unknown variables (for the left-hand-side of the circuit).

   For each resistor:

   $5C_i - V = 5^{3-i} R I_i$

   except the last:

   $5C_0 - V = 75 R I_0$

   Kirchhoff's current law (we can assume that the leg to the op amp is carrying zero current):

   $\sum_{i=0}^{3} I_i = 0$

2. (10 pts) Derive an equation for $V$.

$$\begin{aligned}
\frac{5C_0 - V}{75R} + \sum_{i=1}^{3} \frac{5C_i - V}{5^{3-i}R} &= 0 \qquad (1)\\
\frac{5C_0}{75} + \sum_{i=1}^{3} \frac{5C_i}{5^{3-i}} &= \frac{V}{75} + \sum_{i=1}^{3} \frac{V}{5^{3-i}}\\
5\left(\frac{C_0}{75} + \sum_{i=1}^{3} \frac{C_i}{3^{3-i}}\right) &= V\left(\frac{1}{75} + \sum_{i=1}^{3} \frac{1}{5^{3-i}}\right)\\
V &= 5\left(\frac{C_0}{75} + \sum_{i=1}^{3} \frac{C_i}{5^{3-i}}\right) \Big/ \left(\frac{1}{75} + \sum_{i=1}^{3} \frac{1}{5^{3-i}}\right)\\
&= \frac{5}{94}(C_0 + 3C_1 + 15C_2 + 75C_3)
\end{aligned}$$

3. (10 pts) Assume that $R_a = 100\Omega$ and $R_b = 300\Omega$. Show an equation for $V_{out}$ in terms of $V$.

   Assume $V_-$ is the input to the negative side of the op amp.

   Our voltage divider gives us:

$$V_- = V_{out}\frac{R_a}{R_a + R_b}$$

   The op amp gives us:

$$V_{out} = \beta(V - V_-)$$

   Combining gives us:

$$\begin{aligned}
V_{out} &= \beta\left(V - V_{out}\frac{R_a}{R_b + R_a}\right)\\
V_{out}\left(1 + \beta\frac{R_a}{R_b + R_a}\right) &= \beta V\\
V_{out} &= \frac{\beta V}{1 + \beta\frac{R_a}{R_b + R_a}}
\end{aligned}$$

For large $\beta$, this gives us:

$$\begin{aligned} V_{out} &\approx \frac{R_b + R_a}{R_a} V \\ &= 4V \end{aligned}$$

4. (10 pts) Show $V_{out}$ as a function of the binary number $C$. (show a graph)