Decision Trees

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Regression / Classification So Far…

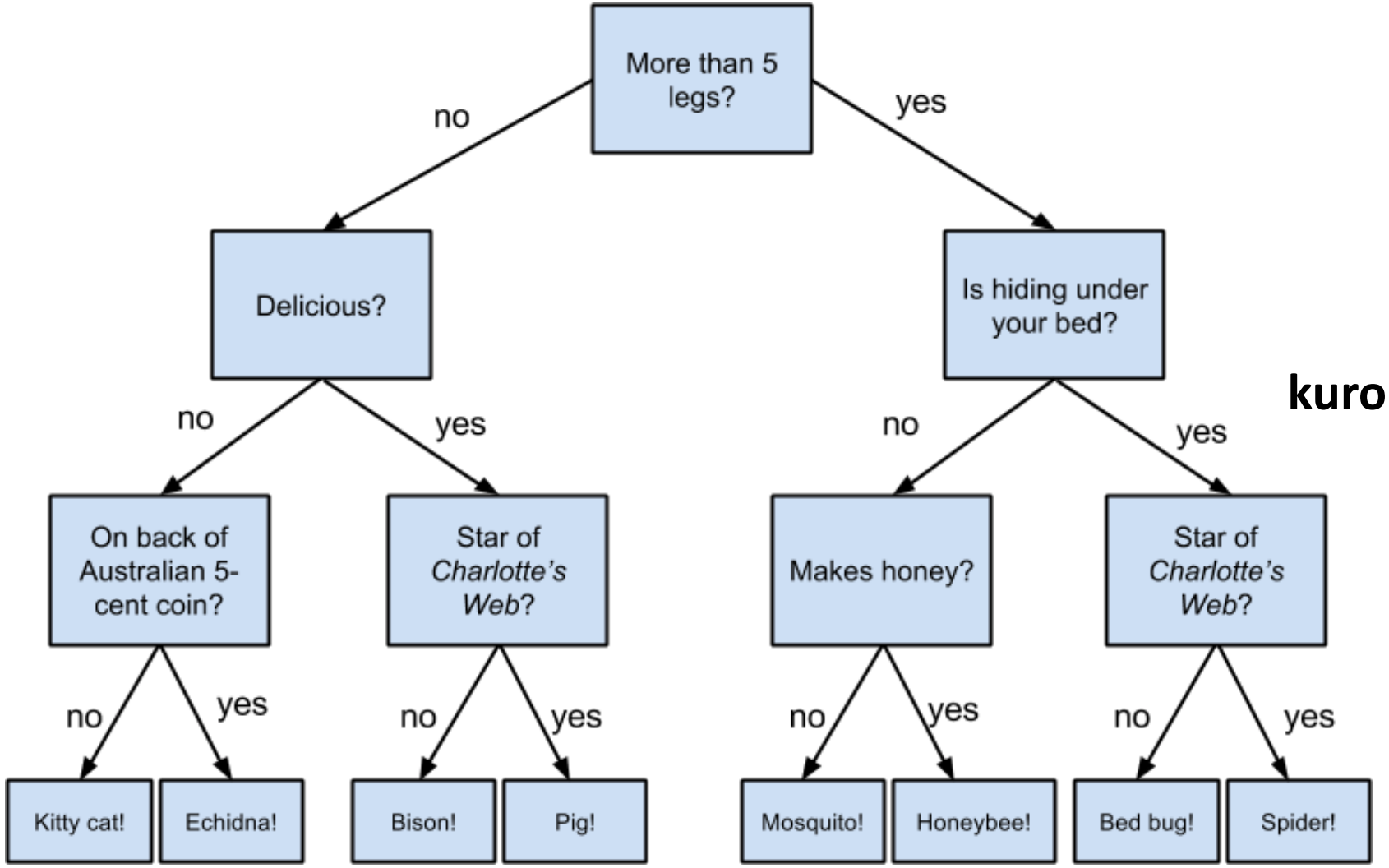
- Input features must be numeric
  - Categorical variables required us to first transform them into numeral variables (via one-hot-encoding)
- Models have been based on continuous functions
- When we have many input features, it is hard to understand how the model works by just looking at the model parameters

Decision Trees

A new way of looking at model representation:

- Ask a recursive set of binary (or N-ary) questions
- Questions refer to specific input features
  - Input features can be numerical or categorical (enumerated data type)
- After the questions: produce a prediction

Decision Tree Example



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Decision Trees

Tree structure:

- A query starts at the root of the tree (the root node)
- A question node asks something specific about a feature in the query
- Depending on the answer, the query “falls down” one of the branches from the question
  - Often binary trees: we have “Yes” or “No” branches
- Each branch can contain additional questions (and branches)
- All paths end in leaf nodes, where the predictions are made

IPAD_M11_L01b

- Partition a feature space
- Example with 2 DOF
- Show tree
- Show sorting of queries
- Example: animal tree

Decision Trees

- Each question cuts the a feature subspace into two pieces
  - These cuts are axis-aligned
- Sequences of questions along a path to a leaf node are Boolean AND operators
- Branches are OR operators
- A tree sorts a set of queries into different leaf nodes

Types of Leaf Node Predictions

Different types of trees make different types of predictions:

- Standard tree: predict a single class
- Probability tree: predict a probability distribution over possible classes
- Regression tree: predict a continuous value

Tree Learning

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Tree Learning

Incremental, greedy algorithm

- Start with an empty tree
- Grow tree by adding ones new question + leaves
- Recompute the predictions
- Repeat

Generic Tree Learning Algorithm

- Training set: feature vectors + correct answer
- Initialize tree with root node and a leaf
  - All training samples are sorted into this leaf
  - Make a prediction that yields the “best” performance

Generic Tree Learning Algorithm

Until an adequate tree is learned:

- Pick best leaf node to replace with
  - A question
  - A pair of leaves (or more)
- Pick the best question to ask
  - Which feature?
  - Which value (or threshold value?)
- Pick the best prediction for each of the new leaves

Generic Tree Learning Algorithm

Choosing the best next question

- With a small, finite set of discrete features, each with a small number of possible values, it is possible to consider all possible questions when evaluating the “best”
- More generally, we will sample from the possible question set
  - Training process becomes a stochastic one!

Probability Tree Learning

- Desired output: class
- Leaf nodes: probability distribution over the possible classes
- General idea: want each of the leaf nodes to contain a “pure” set of training examples

Probability Tree Learning (Intuition)

- Want leaf nodes to be as pure as possible
- Greedy algorithm:
  - Pick the leaf node with the highest impurity to expand
  - Pick the feature and dividing line that best distinguishes the classes
- The greedy algorithm can keep going to an extreme
  - This is the overfitting problem again!

Formalizing Probability Tree Learning

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Formalizing Probability Tree Learning

Measuring purity of leaf nodes:

- Information content
- Gini Impurity

Information vs Gini

Very similar metrics

- Really is an empirical question as to which one to use
- Gini:
  - Less computation
  - Tries to place most frequent class into one of the main branches
- Information:
  - Better balance of trees

Combatting Overfitting

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Model Parameters

- In our modeling work so far, each model had a fixed number of parameters
- We have also discussed the fact that as the complexity of a model increases, we often need more training data to effectively “tie down” the parameters
  - Otherwise, we run the risk of overfitting

Decision Tree Learning

- With trees, we need a distinct set of parameters for:
  - Each question (which feature & which threshold)
  - Each leaf node (e.g., the probability function)
- This means that, as the tree grows, the number of parameters increases
- In fact, a tree can grow beyond the structure that a training set can provide…

Tree Learning

We can allow our tree learning algorithm to execute until all leaves have zero entropy

- Most extreme case: one leaf per training set sample
- Effectively have a custom rule for every training sample

- Very brittle: the regions defined by the tree do not necessarily generalize to independent queries

Combatting Overfitting in Trees

Fundamental idea: constrain the complexity of the tree

- But: what is the most effective way to do this?

Regularization in Trees

Different tree learning algorithms make different choices.
Key ideas:

- Limit the maximum depth of the tree
  - At the limit, forces balanced trees
- Limit the number of leaf nodes
  - Allows more unbalanced trees

Regularization in Trees II

Sample-driven decisions for splitting a leaf node:

- Require a minimum number of samples in a leaf node before allowing it to be expanded
- Proposed split must result in a measurable improvement in performance. Possibilities:
  - Entropy change; Gini Score
  - Likilihood Ratio test
  - Crisp classification: Chi-squared test

Example: Probability Tree Learning

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Example: Probability Tree Learning

- 2D classification problem (from SVM data)
- Baby action recognition
  - Adjust the positive examples:
    - Samples leading up to event are now considered positive
    - Dropping samples immediately after events

Example: Probability Tree Learning

Live example

Regression Trees

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Regression Trees

Leaf nodes output a continuous value:

- Simple (most common) form: samples that fall into a particular leaf node are assigned the same value (i.e., constant function)
  - Yields a piecewise-constant function
- More general case: output is some function of the full feature vector
  - Each leaf node has its own function
  - So, more expressive than using the function over the entire training set

Example: Regression Trees

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Regression Trees

Live example